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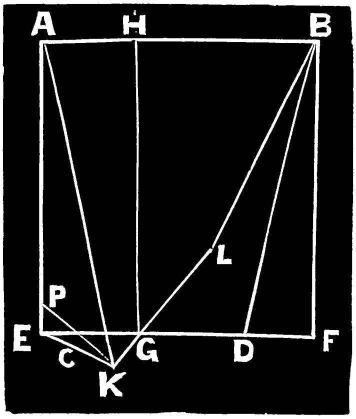
Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $AB=a$, $CD=b=KL$, $AE=HG=BF=h$, $\angle CGK=\angle LGD=\theta$, $AC=AK=l$, $BD=BL=l'$. Let T, T_1 be the vertical tensions at C and D, $CG=KG=x$, $GD=GL=y$. Then $T+T_1=W$, $T-T_1=nW$.



$\therefore T=\frac{1}{2}W(1+n)$, $T_1=\frac{1}{2}W(1-n)$.

$\therefore \frac{1}{2}W(1+n)x=\frac{1}{2}W(1-n)y$ and $x+y=b$.

$\therefore x=\frac{1}{2}b(1-n)$, $y=\frac{1}{2}b(1+n)$.

Similarly, $AH=EG=\frac{1}{2}a(1-n)$,

$HB=FG=\frac{1}{2}a(1+n)$.

Let $EAK=\phi$, $LBF=\psi$.

Q=the momental resistance at K.

Q'=the momental resistence at L.

Draw KP perpendicular to AE. Then $AP=l\cos\phi$.

$PK=l\sin\phi=\sqrt{(EG^2+KG^2-2EG.KG\cos\theta)}$.

$\therefore l\sin\phi=\frac{1}{2}(1-n)\sqrt{(a^2+b^2-2ab\cos\theta)}$.....(1).

Also $Qd\theta=-\frac{1}{2}W(1+n)d(l\cos\phi)=\frac{1}{2}W(1+n)l\sin\phi d\phi$.

$\therefore Qd\theta=\frac{1}{2}W(1+n)l\sin\phi d\phi$.....(2).

From (1), $l\cos\phi d\phi=\dfrac{ab(1-n)\sin\theta d\theta}{2\sqrt{(a^2+b^2-2ab\cos\theta)}}$.....(3).

Multiplying (1) by (3), we get

$$l^2\sin\phi\ \cos\phi\ d\phi=\tfrac{1}{4}ab(1-n)^2\sin\theta\ d\theta\text{.....(4)}.$$

Eliminating $d\theta$, $d\phi$ between (2) and (4) we get

$$Q=\tfrac{1}{8}(1-n^2)(1+n)W(ab/l\cos\phi)\sin\theta.$$

Similarly, $$Q'=\tfrac{1}{8}(1-n^2)(1+n)W(ab/l'\cos\psi)\sin\theta.$$

Since the deflection is small we can place $l\cos\phi=l'\cos\psi=h$.

$\therefore Q=\frac{1}{8}(1-n^2)(1-n)W(ab/h)\sin\theta$, $Q'=\frac{1}{8}(1-n^2)(1+n)W(ab/h)\sin\theta$.

Momental resistance$=Q+Q'$.

$\therefore Q+Q'=\frac{1}{8}(1-n^2)(1-n+1+n)W(ab/h)\sin\theta=\frac{1}{4}(1-n^2)W(ab/h)\sin\theta$.

AVERAGE AND PROBABILITY.

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152. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

A square hole is cut through the center of a sphere of radius r. Show that the average volume removed is $\frac{1}{12}r^3(23\sqrt{2}-28)$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $x^2+y^2+z^2=r^2$ be the equation to the sphere, $2b$=side of square hole.

Volume removed$=16\displaystyle\int_0^{\frac{1}{4}\pi}\int_0^{b\sec\theta} z\rho d\theta d\rho$. $z=\sqrt{(r^2-\rho^2)}$.

The limits of b are 0 and $\frac{1}{2}r\sqrt{2}=b'$. Let average volume$=\triangle$.

$$\therefore \triangle = 16\int_0^{\frac{1}{4}\pi}\int_0^{b'}\int_0^{b\sec\theta}\sqrt{(r^2-\rho^2)}\rho d\theta db d\rho \Big/ \int_0^{b'} db$$

$$=\frac{16\sqrt{2}}{3r}\int_0^{\frac{1}{4}\pi}\int_0^{b'}\left[r^3-(r^2-b^2\sec^2\theta)^{\frac{3}{2}}\right]d\theta db$$

$$=\frac{r^3\sqrt{2}}{3}\int_0^{\frac{1}{4}\pi}\left[8\sqrt{2}-(5-\sec^2\theta)\sqrt{(2-\sec^2\theta)}-6\cos\theta\sin^{-1}\left(\frac{\sec\theta}{\sqrt{2}}\right)\right]d\theta$$

$$=\frac{r^3\sqrt{2}}{3}\left[2\pi\sqrt{2}-\tfrac{3}{2}\pi\sqrt{2}-\int_0^{\frac{1}{4}\pi}\left((5-\sec^2\theta)\sqrt{(2-\sec^2\theta)}-\frac{6\tan^2\theta}{\sqrt{(2-\sec^2\theta)}}\right)d\theta\right].$$

Let $\tan\theta=\sin\phi$.

$$\therefore \triangle=\frac{r^3\sqrt{2}}{3}\left[\tfrac{1}{2}\pi\sqrt{2}-\int_0^{\frac{1}{2}\pi}\frac{4\cos^2\phi-\sin^2\phi\cos^2\phi-6\sin^2\phi}{1+\sin^2\phi}d\phi\right]$$

$$=\frac{r^3\sqrt{2}}{3}\left[\tfrac{1}{2}\pi\sqrt{2}-\int_0^{\frac{1}{2}\pi}\left(\sin^2\phi-12+\frac{16}{1+\sin^2\phi}\right)d\phi\right]=\frac{\pi r^3}{12}(23\sqrt{2}-28).$$

Also solved by the Proposer.

GROUP THEORY.

2. **Proposed by W. BURNSIDE, The Croft, Bromley Road, Catford, England.**

Show that the group of the biquadratic equation $x^4+2ax^2+b=0$, in which a and b are rational numbers, while a^2-b is not the square of a rational number, is in general a dihedron group of order 8; but that (*i*) if b is the square of a rational number the group is a non-cyclical Abelian group of order 4; and (*ii*) if $b=a^2\div(1+n^2)$, where n is a rational number, the group is a cyclical group of order 4.

Solution by L. E. DICKSON, Ph. D., The University of Chicago.

The problem may be solved most readily by making use of the simple expressions resulting for the roots of this special quartic. To obtain a more instructive example of the application of groups to equations, I treat the problem without assuming the quartic to be solvable by radicals.

Consider the general quartic $x^4+ax^3+bx^2+cx+d=0$, and call its roots x_1, x_2, x_3, x_4. As is well known, the functions

$$y_1=x_1x_2+x_3x_4,\quad y_2=x_1x_3+x_2x_4,\quad y_3=x_1x_4+x_2x_3$$

are the roots of the cubic resolvent

$$y^3-by^2+(ac-4d)y-a^2d+4bd-c^2=0.$$

For our special quartic, this resolvent becomes